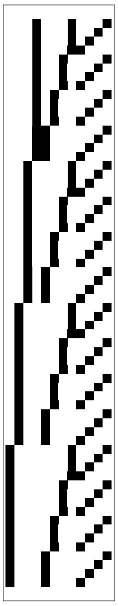
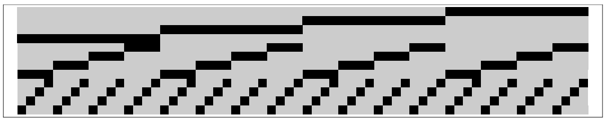
```
(* 10^6-
  Fold  Increase  in  Computation  Speed  of  a  Scroll  Matrix  Pseudoinverse *)
(* Copyright   Doug Youvan   May 29,
 2006    www.youvan.com    www.pseudocolor.com *)

(* 64 codons in binary 'scroll matrix'
  in canonical order as defined by the following *)

t =.; g =.; c =.; a =.;
nucgc = Tuples[{a, c, g, t}, 3];
a = {0, 0, 0, 1};
c = {0, 0, 1, 0};
g = {0, 1, 0, 0};
t = {1, 0, 0, 0};
nucgc = Flatten[nucgc];
nucgc = Partition[nucgc, 12];
(* end of definition *)
(* this could also be seen as a unique mapping of alphanumerics onto a series
  of the powers of base 2; in this particular case of an alphabet of a,
 c, g, t the mapping is onto 1, 2, 4, 8 in base 2 *)
ArrayPlot[nucgc]
```



- Graphics -

```
(* take pseudoinverse ~ 4 minutes on Sony VAIO PCG-K64S *)

date1 = Date[];
pin = PseudoInverse[nucgc];
date2 = Date[];
timeforpin = date2 - date1
ArrayPlot[pin]
```

```
{0, 0, 0, 0, 5, 14.5523040}
```



```
- Graphics -
```

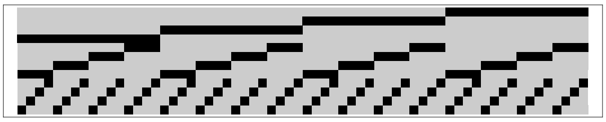
```
(* for a scroll matrix, reproduce pseudoinverse by using only transpose
  and solving the following equations requiring only 30 ms of time *)

trana =.;
tranb =.;
date3 = Date[];
Solve[pin == ((Transpose[nucgc] * trana) - tranb), {trana, tranb}]
date4 = Date[];
timeforsolve = date4 - date3
```

$\left\{\left\{\text{trana} \to \frac{1}{16}, \text{tranb} \to \frac{1}{96}\right\}\right\}$

```
{0, 0, 0, 0, 0, 0.0300432}
```

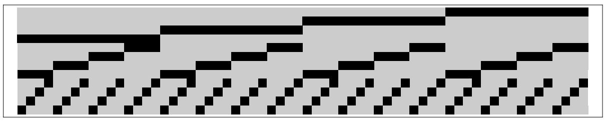
```
(* make substitions *)

date5 = Date[];
ma = ((Transpose[nucgc] / 16) - 1 / 96);
date6 = Date[];
timeforma = date6 - date5

(* check if the pseudoinverse 'pin' generated by the Mathematica SVD
  PseudoInverse function is the same as the transpose solution for 'ma' *)

ArrayPlot[ma]

ma == pin (* ? *)
```

```
{0, 0, 0, 0, 0, 0.0100144}
```



```
- Graphics -

True
```

```
(* Test scroll method' s speed by converting the scroll
  matrix ' nucgc' to ' nucgcns' by dropping three rows: 57,51,49,
 yielding a 61 x12 matrix; then comment out so it does not rerun;
 this takes about 2.2 hrs as it did in Example 17 *)

(*
 nucgcns=Drop[nucgc,{57}];
 nucgcns=Drop[nucgcns,{51}];
 nucgcns=Drop[nucgcns,{49}];

 date8=Date[];
 pinns=PseudoInverse[nucgcns];
 date9=Date[];
 timeforpin=date9-date8

 (* {0,0,0,2,-47,26.1457264`8.868945656625751} *)   *)

ArrayPlot[pinns]
```



```
- Graphics -
```

```
(* check how scroll method works on simpler scroll matrices *)

simple1 =
  {{0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1},
   {0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0},
   {0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0},
   {0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0}};

pis1 = PseudoInverse[simple1]
```

$$\left\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\left\{\frac{1}{9},\frac{1}{9},\frac{1}{9},\frac{1}{9}\right\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\left\{\frac{1}{9},\frac{1}{9},\frac{1}{9},\frac{1}{9}\right\},\left\{-\frac{2}{9},-\frac{2}{9},-\frac{2}{9},\frac{7}{9}\right\},\left\{-\frac{2}{9},-\frac{2}{9},\frac{7}{9},-\frac{2}{9}\right\},\left\{-\frac{2}{9},\frac{7}{9},-\frac{2}{9},-\frac{2}{9}\right\},\left\{\frac{7}{9},-\frac{2}{9},-\frac{2}{9},-\frac{2}{9}\right\}\right\}$$

```
simple2 =
  {{0, 1,  0, 1},
   {0, 1,  1, 0},
   {1, 0,  0, 1},
   {1, 0,  0, 1}};

pis2 = PseudoInverse[simple2]
```

$$\left\{\left\{-\frac{1}{2},\frac{1}{4},\frac{3}{8},\frac{3}{8}\right\},\left\{\frac{1}{2},\frac{1}{4},-\frac{1}{8},-\frac{1}{8}\right\},\left\{-\frac{1}{2},\frac{3}{4},\frac{1}{8},\frac{1}{8}\right\},\left\{\frac{1}{2},-\frac{1}{4},\frac{1}{8},\frac{1}{8}\right\}\right\}$$

```
simple3 =
  {{0, 0, 1,  0, 0, 1},
   {0, 0, 1,  0, 1, 0},
   {0, 0, 1,  1, 0, 0},
   {0, 1, 0,  0, 0, 1},
   {0, 1, 0,  0, 1, 0},
   {0, 1, 0,  1, 0, 0},
   {1, 0, 0,  0, 0, 1},
   {1, 0, 0,  0, 1, 0},
   {1, 0, 0,  1, 0, 0}};
```

```
pis3 = PseudoInverse[simple3]
```

$$\Big\{\Big\{-\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, \frac{5}{18}, \frac{5}{18}\Big\},$$
$$\Big\{-\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, \frac{5}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}\Big\},$$
$$\Big\{\frac{5}{18}, \frac{5}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}\Big\},$$
$$\Big\{-\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}\Big\},$$
$$\Big\{-\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}\Big\},$$
$$\Big\{\frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}\Big\}\Big\}$$

```
Total[Flatten[simple3]]
```

18

```
Flatten[pis3, 3]
```

$$\Big\{-\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, \frac{5}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, \frac{5}{18},$$
$$\frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, \frac{5}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18},$$
$$-\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18},$$
$$-\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}\Big\}$$

```
Total[Flatten[pis3, 3]] (* alphabet size = 3 ? *)
```

3

```
(* test for invariance by randomizing rows of simple3 to make simple31 *)

simple31 =
  {{0, 0, 1, 0, 0, 1},
   {0, 1, 0, 0, 0, 1},
   {1, 0, 0,  0, 1, 0},
   {0, 1, 0,  0, 1, 0},
   {0, 1, 0,  1, 0, 0},
   {1, 0, 0, 0, 0, 1},
   {0, 0, 1,  0, 1, 0},
   {0, 0, 1,  1, 0, 0},
   {1, 0, 0,  1, 0, 0}};

pis31 = PseudoInverse[simple31]
```

$$\left\{\left\{-\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}\right\},\right.$$
$$\left\{-\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, \frac{5}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}\right\},$$
$$\left\{\frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, \frac{5}{18}, -\frac{1}{18}\right\},$$
$$\left\{-\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, \frac{5}{18}\right\},$$
$$\left\{-\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}\right\},$$
$$\left.\left\{\frac{5}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}, \frac{5}{18}, -\frac{1}{18}, -\frac{1}{18}, -\frac{1}{18}\right\}\right\}$$

```
simple3 == simple31
pis3 == pis31
Total[Flatten[simple3]] == Total[Flatten[simple31]]
Total[Flatten[pis31, 3]] == Total[Flatten[pis31, 3]]
Min[pis3] == Min[pis31]
Max[pis3] == Max[pis31]
Max[pis3] - Min[pis3] == Max[pis31] - Min[pis31]
```

False

False

True

True

True

True

True

```
MatrixForm[pis3]
ma = 0.
ma = ((Transpose[simple3] * 6) - 1) / 18;
MatrixForm[ma]
ma == pis3
```

$$\begin{pmatrix} -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & \frac{5}{18} & \frac{5}{18} \\ -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & \frac{5}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} \\ \frac{5}{18} & \frac{5}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} \\ -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} \\ -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} \\ \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} \end{pmatrix}$$

```
0.
```

$$\begin{pmatrix} -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & \frac{5}{18} & \frac{5}{18} \\ -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & \frac{5}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} \\ \frac{5}{18} & \frac{5}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} & -\frac{1}{18} \\ -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} \\ -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} \\ \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} & \frac{5}{18} & -\frac{1}{18} & -\frac{1}{18} \end{pmatrix}$$

```
True
```

```
(* This method has been used (data and code not shown)
  for a mapping of a physicochemical property of the amino acids
  (hydropathy values) as in Example 17 by padding the stop codons at 49,
 51, 57 in the 64 x 20 matrix with an average value of
    that property.  This obviates the need for an SVD-
   based pseudoinverse such that this scroll method can be used with a 10^6 -
   fold increase in computational speed as compared with the non-
   padded 61 x 20 matrix that requires SVD. Changes to the bottom figure
    in Example 17 are < ~ 5 % and not very noticable within the scatter.  *)
```